

# 演習答え：シンプソンの公式

(1)  $n=2$ のLagrangeの補間公式

$$L_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f_2$$

(2)  $L_2(x) = c_0 f_0 + c_1 f_1 + c_2 f_2$  とすると

$$c_0 = \int_0^2 \frac{(x-1)(x-2)}{(0-1)(0-2)} dx = \int_0^2 \frac{x^2 - 3x + 2}{2} dx = \left[ \frac{x^3}{6} - \frac{3x^2}{4} + x \right]_0^2 = \frac{1}{3}$$

$$c_1 = \int_0^2 \frac{(x-0)(x-2)}{(1-0)(1-2)} dx = \int_0^2 (-x^2 + 2x) dx = \left[ -\frac{x^3}{3} + x^2 \right]_0^2 = \frac{4}{3}$$

$$c_2 = \int_0^2 \frac{(x-0)(x-1)}{(2-0)(2-1)} dx = \int_0^2 \frac{x^2 - x}{2} dx = \left[ \frac{x^3}{6} - \frac{x^2}{4} \right]_0^2 = \frac{1}{3}$$

よって3つの積分点の重みが 1 : 4 : 1